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STELLAR EVOLUTION IN BLUE POPULOUS CLUSTERS OF THE SMALL MAGELLANIC CLOUD AND THE PROBLEMS OF ENVELOPE SEMICONVECTION AND CONVECTIVE CORE OVERSHOOTING

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ABSTRACT

Two of the blue populous clusters in the Small Magellanic Cloud, NGC 330 and NGC 458, contain a considerably larger number of evolved stars than are found in comparably young clusters in our Galaxy. This richness makes them valuable tools for studying post–main-sequence evolution in the mass range 4–15 $M_{\odot}$. Accordingly, new theoretical evolutionary sequences of models for stars with low metallicities, appropriate to the Small Magellanic Cloud, are derived here with both standard Cox-Stewart opacities and the new Rogers-Iglesias opacities. We find that only those sequences with little or no convective core overshooting can reproduce the two most critical observations: the maximum effective temperature displayed by the hot evolved stars and the difference between the average bolometric magnitudes of the hot and cool evolved stars. An upper limit to the ratio of the mean overshoot distance beyond the classical Schwarzschild core boundary to the local pressure scale height can be set at $d/H_P < 0.2$. Although the use of the maximum effective temperature is subject to certain mild restrictions, the bolometric magnitude difference is an ideal quantity, being sensitive only to the stellar core mass. A second important result derived here from the frequency of cool supergiants in NGC 330 is that the Ledoux criterion (rather than the Schwarzschild criterion) for convection and semiconvection in the envelopes of massive stars is strongly favored. Both conclusions are independent of current uncertainties about the adopted stellar metallicities and the radiative opacities. However, the Rogers-Iglesias opacities produce superior model fits to other observational quantities. Residuals from the fitting for NGC 330 suggest the possibility of fast interior rotation in the stars of this cluster. NGC 330 and NGC 458 turn out to have ages of $\sim 3 \times 10^7$ yr and $\sim 1 \times 10^8$ yr, respectively.

Subject headings: convection — Magellanic Clouds — open clusters and associations: individual (NGC 330, 458) — stars: evolution

1. INTRODUCTION

Star clusters are natural laboratories for the observational and theoretical study of stellar evolution off the main sequence. In considering massive stars, however, it is necessary to resort to young and intermediate-age clusters, which in our Galaxy are too poorly populated, in all known cases, to be of much use individually. Pooling their data, of course, conveys more information. Yet the total number of evolved stars available is still small, and these stars are inevitably heterogeneous in age and initial chemical composition. Therefore, it is useful to examine the more populous star clusters of the Magellanic Clouds, of which the best examples in the Small Cloud are NGC 330 (Arp 1959b; Robertson 1974; Carney, Janes, & Flower 1985, hereinafter CJF) and NGC 458 (Arp 1959a).

Three important questions that might be answered in part from a combined observational and theoretical study of these two clusters are the following: (1) the precise metals abundance of young objects in the metal-poor Small Magellanic Cloud, (2) the proper criterion to use for convection and semiconvection in massive stars, and (3) the distance of convective overshooting of turbulent elements beyond the classical convective core boundary. In their specific investigations of NGC 330, CJF and Robertson (1973) tried to answer the first and second questions, respectively, but they obtained only ambiguous results. From the more complete theoretical study presented here, it is possible to answer all three questions with a fair degree of certainty.

2. NEW THEORETICAL EVOLUTIONARY SEQUENCES

New stellar models with very low metallicities have been computed to cover the phases of evolution from the zero-age main sequence to the end of core helium burning. The input physics used is the same as in Chin & Stothers (1991), except that in addition to the standard opacities of Cox & Stewart (1965, 1970) we have now also used the new opacities of Rogers & Iglesias (1992) with their increased metal-line contribution (see also Iglesias & Rogers 1991). Thus, two parallel sets of model sequences were computed.

Free parameters for our primary grid of sequences are assigned as follows: stellar mass, $M/M_{\odot} = 3$, 5, 7, 10, and 15 $M_{\odot}$; initial helium abundance by mass, $Y_e = 0.24$; initial metals abundance by mass, $Z_e = 0.002$ and 0.004; ratio of the convective mixing length to the local pressure scale height in the outer convection zone, $\alpha_P = 2$; and ratio of the convective overshoot distance for effective mixing beyond the classical Schwarzschild core boundary to the local pressure scale height, $d/H_P = 0$, 0.20, and 0.35. Semiconvective mixing in the intermediate layers of the star has been calculated according to either the Schwarzschild (temperature-gradient) criterion or the Ledoux (density-gradient) criterion for convective neutrality. Stellar wind mass loss from stars with such low metals abundances is uncertain, but is probably less than that observed for Galactic stars with solar metals abundances; accordingly, two parallel sets of model sequences were computed here, comprising the case of (a) no mass loss and (b) mass loss at a rate appropriate to Galactic stars, as given by the fitted formula of Nieuwenhuijzen & de Jager (1990). Finally, axial rotation has been ignored, but its effect on the evolutionary track is expected to be negligible (Endal & Sofia 1976) unless the initial rotational state of the star is near critical (Kippenhahn, Meyer-Hofmeister, & Thomas 1970; Meyer-Hofmeister 1972).

Our two choices for the initial metals abundance will be discussed in § 3. Because the properties of the stellar models are not very sensitive to minor changes in α_P, we have assigned a blanket value of $\alpha_P = 2$, which provides a rough match between the effective temperatures of our cool evolved models and the effective temperatures of observed red giants and supergiants in NGC 330 and NGC 458. One set of model sequences for $Z_e = 0.004$ has been recomputed with $\alpha_P = 1$, yielding a Hayashi line in the Hertzsprung-Russell (H-R) diagram that is cooler by 0.12 dex in log T_e; however, the tips of the blue loops remain fixed in effective temperature within ± 0.003 dex. Overshooting from the bottom of the outer convection zone has been ignored, because it has a negligible effect on the properties of a developed blue loop if the overshoot distance is less than $\sim 0.4 H_P$ (Matraka, Wassermann, & Weigert 1982; Stothers & Chin 1991a; Alongi et al. 1991).

If convective core overshooting is assumed to be unimportant, post–main-sequence evolution at the two highest masses (10 and 15 $M_\odot$) depends critically on the criterion for envelope convection and semiconvection and, to a lesser extent, on the opacities that are adopted. In the case of the Schwarzschild criterion, the star leaves the main sequence and burns all but the last of the helium in its core as a blue supergiant, while if the Ledoux criterion is adopted the star stays blue only for very low envelope opacities—as represented here by the Cox-Stewart opacities with $Z_e = 0.002$ (see also Trimble, Paczyński, & Zimmerman 1973; Alcock & Paczyński 1978; Dearborn & Trimble 1980; Brunish & Truran 1982; Baraffe & El Eid 1991). In all other cases, a star with a mass of 15 $M_\odot$ or less evolves first to the red giant or supergiant region; then it may or may not execute a blue loop toward higher effective temperatures.

Convective core overshooting on the main sequence increases the mass of the helium core and thereby raises the luminosity of the star. Of particular interest here if the star has a blue loop during the subsequent core helium-burning phase are two additional effects: (1) a reduction of the effective temperature at the tip of the blue loop and (2) a relative brightening of the star's average red branch luminosity compared to its average blue loop luminosity. It is in fact primarily overshooting during the main-sequence phase of evolution that has evolutionary significance; overshooting from the helium-burning convective core exerts relatively little influence on the evolution of stars as massive as those considered here (Chin & Stothers 1991). The reduction in the ratio of the lifetime of core helium burning to the lifetime of core hydrogen burning is also potentially observable, but not with the presently available data for NGC 330 and NGC 458.

Selected evolutionary sequences that produce a red giant or supergiant at the onset of core helium depletion are summarized in Tables 1, 2, 3, and 4. Corresponding tracks in the H-R diagram are plotted in Figures 1, 2, and 3 for the evolutionary models without mass loss, since the effect of mass loss is very small. The stellar lifetimes listed in the tables refer to the following phases: core hydrogen burning, τ_H; core helium burning, τ_{He}; and the blue phase, τ_b, and the red phase, τ_r, of core helium burning. The terms "blue tip" and "red branch" refer to the characteristic regions of the H-R diagram that the star occupies during the *slow* stages of core helium depletion. The dividing line is set at log $T_e = 3.82$. Since neutrino losses drastically accelerate the star's evolution after central helium exhaustion (Stothers 1969, 1985), the very late phases of evolution can be ignored for the purposes of this paper.

3. INITIAL CHEMICAL COMPOSITION

3.1. *NGC 330*

Abundance determinations made on the bright stars in NGC 330 apply almost exclusively to the red supergiants. The metals observed, however, are only a representative sample of all the metals, and their measured abundances relative to hydrogen are expressed as $[m/H] = \log [N(m)/N(H)]_* - \log [N(m)/N(H)]_\odot$. Intermediate-band photometry, which measures primarily CN, seems to indicate $[m/H] \approx -1.8$ (Richtler & Nelles 1983; CJF), while CO band photometry (McGregor & Hyland 1984) and broad-band infrared photometry (CJF) indicate, rather more inclusively, $[m/H] \approx -1.0$. Strömgren photometry suggests $[m/H] \approx -1.2$ (Grebel & Richtler 1992). Spectroscopic analyses of two red supergiants in the cluster also yield $[m/H] \approx -1.0$ (Spite et al. 1986; Barbuy et al. 1991; Spite, Richtler, & Spite 1991) or -0.8 (Bessell 1991b). Very low metallicity is apparent, too, in the relatively early spectral types, G2–G6 Ib, displayed by these red stars (Feast 1979). A spectroscopic analysis of one B-type member—a giant—has yielded $[m/H] \approx -1.0$, and also suggests a "normal" helium abundance (Reitermann et al. 1990).

TABLE 1

EVOLUTIONARY SEQUENCES BASED ON THE LEDOUX CRITERION AND ON THE COX-STEWART OPACITIES WITH $Z_e = 0.004$

					Blue Tip		Red Branch
$M/M_\odot$	d/H_P	$\tau_H(10^6$ yr)	τ_{He}/τ_H	τ_b/τ_r	log T_e	log $L/L_\odot$	log $L/L_\odot$
3	0	207.233	0.243	0.367	3.872	2.611	2.308–2.358
	0.35	268.361	0.091	0.000	...	...	2.711–2.875
5	0	69.198	0.171	2.286	4.024	3.408	3.133–3.148
	0.35	88.895	0.076	0.392	3.890	3.602	3.510–3.619
7	0	36.836	0.142	5.204	4.114	3.880	3.646–3.657
	0.35	46.568	0.070	1.084	3.999	4.071	3.995–4.091
10	0	20.590	0.112	6.126	4.181	4.348	4.182–4.189
	0.35	25.420	0.063	1.521	4.033	4.508	4.460–4.544
15	0	11.801	0.092	6.211	4.217	4.815	4.727–4.740
	0.35	14.130	0.060	0.738	3.998	4.954	4.928–5.004

TABLE 2

EVOLUTIONARY SEQUENCES BASED ON THE LEDOUX CRITERION AND ON THE ROGERS-IGLESIAS OPACITIES WITH $Z_e = 0.002$

$M/M_\odot$	d/H_P	$\tau_H(10^6$ yr)	τ_{He}/τ_H	τ_b/τ_r	Blue Tip log T_e	Blue Tip log $L/L_\odot$	Red Branch log $L/L_\odot$
3	0	251.963	0.213	0.192	3.850	2.535	2.251–2.303
	0.35	322.403	0.080	0.000	...	...	2.663–2.806
5	0	78.563	0.143	2.391	3.995	3.310	3.046–3.054
	0.35	101.026	0.072	0.217	3.851	3.537	3.447–3.552
7	0	39.982	0.135	8.231	4.108	3.847	3.582–3.590
	0.35	51.085	0.065	0.862	3.968	4.011	3.923–4.027
10	0	21.784	0.111	5.556	4.180	4.305	4.137–4.146
	0.35	26.843	0.060	1.187	4.016	4.473	4.417–4.508
15	0	12.155	0.088	3.784	4.187	4.802	4.708–4.720
	0.35	14.537	0.058	0.000	...	...	4.907–4.982

TABLE 3

EVOLUTIONARY SEQUENCES BASED ON THE LEDOUX CRITERION AND ON THE ROGERS-IGLESIAS OPACITIES WITH $Z_e = 0.004$

$M/M_\odot$	d/H_P	$\tau_H(10^6$ yr)	τ_{He}/τ_H	τ_b/τ_r	Blue Tip log T_e	Blue Tip log $L/L_\odot$	Red Branch log $L/L_\odot$
3	0	275.137	0.230	0.000	...	...	2.195–2.271
	0.35	349.188	0.081	0.000	...	...	2.571–2.792
5	0	82.799	0.177	0.538	3.894	3.242	3.004–3.048
	0.35	106.140	0.072	0.000	...	...	3.415–3.534
7	0	41.356	0.142	1.920	4.030	3.777	3.540–3.582
	0.35	52.749	0.065	0.000	...	...	3.905–4.018
10	0	22.192	0.115	1.730	4.102	4.265	4.090–4.129
	0.35	27.428	0.062	0.195	3.863	4.427	4.369–4.496
15	0	12.256	0.091	0.715	4.131	4.765	4.660–4.700
	0.35	14.745	0.059	0.000	...	...	4.879–4.977

TABLE 4

EVOLUTIONARY SEQUENCES BASED ON THE LEDOUX CRITERION AND INCLUDING MASS LOSS

Opacities	Z_e	$M/M_\odot$	d/H_P	Final $M/M_\odot$	$\tau_H(10^6$ yr)	τ_{He}/τ_H	τ_b/τ_r	Blue Tip log T_e	Blue Tip log $L/L_\odot$	Red Branch log $L/L_\odot$
Cox-Stewart	0.004	7	0	6.92	36.954	0.143	3.952	4.106	3.871	3.645–3.660
		10	0	9.79	20.621	0.115	5.447	4.167	4.336	4.174–4.186
		15	0	14.26	11.808	0.090	4.199	4.173	4.793	4.717–4.727
Rogers-Iglesias	0.002	7	0	6.94	40.043	0.134	7.353	4.106	3.833	3.567–3.596
		10	0	9.81	21.867	0.111	4.541	4.162	4.298	4.126–4.148
		15	0	14.21	12.292	0.085	2.718	4.136	4.785	4.706–4.721
Rogers-Iglesias	0.004	7	0	6.92	41.394	0.147	1.764	4.019	3.770	3.538–3.581
		10	0	9.76	22.242	0.116	1.411	4.074	4.248	4.088–4.121
		15	0	13.73	12.315	0.091	0.000	...	...	4.649–4.692

Two blue supergiants similarly show $[m/H] \approx -1.0$ (Barbuy et al. 1991; Spite et al. 1991). These results would indicate an average metal deficiency of a factor of ~ 10 in NGC 330.

On the other hand, NGC 330 not only is one of the youngest clusters in the Small Magellanic Cloud but also is located near the middle of the rotating central bar (Arp 1961) where thorough mixing of interstellar material might be expected to occur. Other cool luminous stars in the Small Cloud, as well as a number of H II regions and supernova remnants, show $[m/H] \approx -0.7$ (Russell & Bessell 1989; Russell & Dopita 1990; Spite & Spite 1990; and references therein). Two B-type field supergiants in the Small Cloud show $[m/H] \leq -0.6$ (Osmer 1973), and a B0.5 V field dwarf likewise shows $[m/H] \approx -0.7$ (Dufton, Fitzsimmons, & Howarth 1990); all three of these B stars apparently contain "normal" helium. This being the case, it would be strange if NGC 330 were really twice as metal-deficient as these other young objects.

Because of this uncertainty, we are forced to consider a range of metals reduction factors, say, a range of 5–10 down from normal (or solar) abundances. If the solar metals abundance is taken to be $Z_e \approx 0.02$, then the metals abundance of NGC 330 lies somewhere between 0.002 and 0.004.

The helium abundance of NGC 330, as mentioned above, appears to be "normal" and therefore can be assumed to be

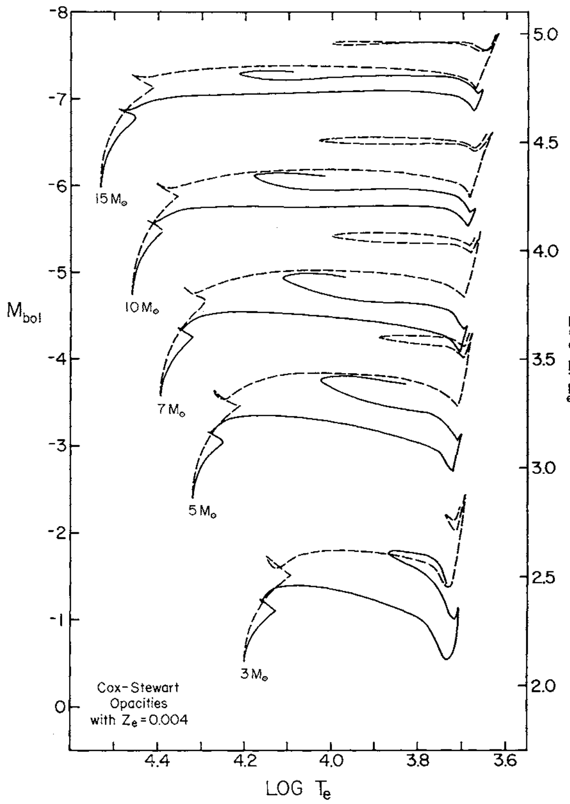

FIG. 1.—Theoretical H-R diagram showing evolutionary tracks based on the Ledoux criterion for convection and semiconvection, in the case of the Cox-Stewart opacities with $Z_e = 0.004$. Mass loss has been ignored. The tracks run from the zero-age main sequence to the end of core helium burning. Solid lines refer to stellar models without convective core overshooting, and dashed lines refer to stellar models with overshooting parameterized by $d/H_P = 0.35$.

the same as that of well-observed H II regions in the Small Cloud, $Y_e = 0.24 \pm 0.01$ (Dufour 1984; Russell & Dopita 1990; and references therein).

3.2. *NGC 458*

No direct measurements of the chemical composition of stars in NGC 458 are available. However, this cluster, like NGC 330, contains red supergiants that show markedly bluer $B-V$ colors than do their Galactic counterparts, a fact which suggests a large metallicity difference (Hagen & van den Bergh 1974). Their higher effective temperatures imply a much lower metals abundance (e.g., Schwarzschild 1958). Since NGC 458 is an outlier in the Small Cloud (Arp 1961), its metals abundance could be as low as NGC 330's. We assume, therefore, that NGC 458 shares the same chemical composition with the average young metal-poor objects in the Small Cloud.

4. ANALYSIS OF THE SUPERGIANTS IN NGC 330

4.1. *Observational Data*

The observed $(V, B-V)$ diagram of NGC 330 contains three separate groups of stars: upper main-sequence stars, blue supergiants, and red supergiants (Arp 1959b; Robertson 1974; CJF). Radial-velocity data (Feast & Black 1980; CJF) confirm the membership of most of the supergiants. It is reasonable, therefore, to assume that most or all of the upper main-sequence stars are members, too. Following CJF's careful study, we adopt their membership list as well as their small correction factor for Robertson's (1974) V magnitudes. Robertson's $B-V$ colors are accepted as they stand.

Seven of the nine blue supergiant members have published spectral types, which range from B5 I to A2 I (Feast 1964; Hyland 1971, in Robertson 1974; CJF). These spectral types correlate well with the supergiants' $B-V$ colors, and follow the expected Galactic relation if the supergiants in NGC 330 are reddened by $E_{B-V} = 0.03$, as CJF have shown. We therefore adopt $(B-V)_0 = B-V - 0.03$ for the two blue supergiants without known spectral types. To transform spectral type or, if necessary, $(B-V)_0$ to effective temperature, the relations tabulated by Fitzpatrick & Garmany (1990) for luminosity class Ib supergiants will be employed. Bolometric absolute magnitudes then follow from the observed V magnitudes by applying, in turn, an extinction correction of $A_V = 0.1$ mag (CJF), a small bolometric correction that is a very weak function of effective temperature (Flower 1977), and a true distance modulus of 18.8 mag (Stothers 1983, 1988). This distance modulus is based on an average of many independent derivations of the zero-point luminosities for Galactic RR Lyrae stars and classical Cepheids, and also contains a small correction for the metal-poor chemical composition of the Small Cloud Cepheids.

Unlike the nine blue supergiants, the 15 red supergiant members of NGC 330 display spectral types that are strongly affected by the cluster's low metallicity (Feast 1979) and so are unsuitable for deriving effective temperatures. Eleven of these red G2–G6 Ib supergiants, however, have published $J-K$

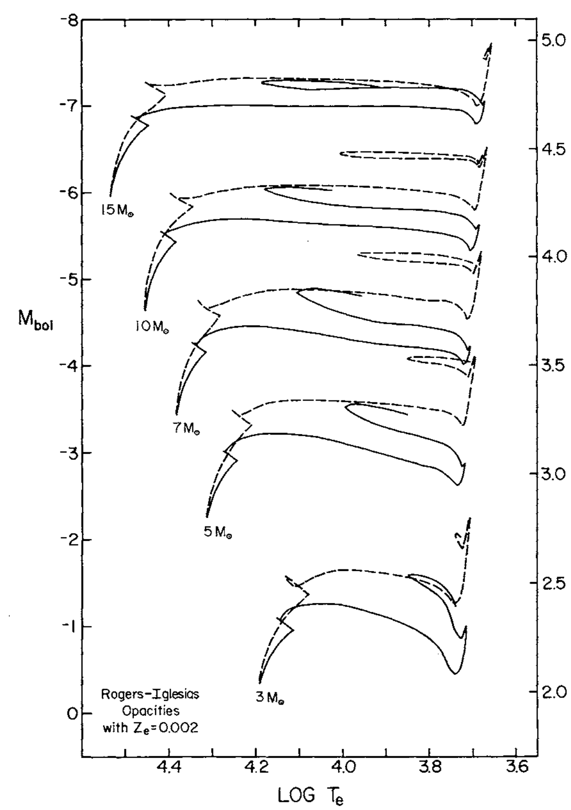

FIG. 2.—Theoretical H-R diagram showing evolutionary tracks based on the Ledoux criterion for convection and semiconvection, in the case of the Rogers-Iglesias opacities with $Z_e = 0.002$. Mass loss has been ignored. Same notation as for Fig. 1.

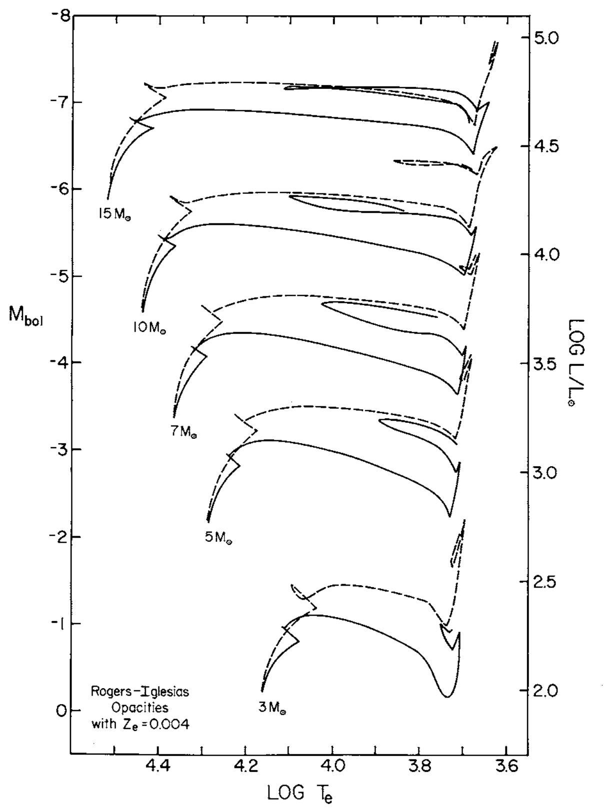

FIG. 3.—Theoretical H-R diagram showing evolutionary tracks based on the Ledoux criterion for convection and semiconvection, in the case of the Rogers-Iglesias opacities with $Z_e = 0.004$. Mass loss has been ignored. Same notation as for Fig. 1.

colors, by means of which reliable effective temperatures and bolometric corrections have been derived by CJF. All but one of the 11 stars define a tight relation between $J-K$ and $B-V$, which can be used to estimate $J-K$ values for the four other red supergiants that only have published $B-V$ colors. Then bolometric absolute magnitudes follow as for the blue supergiants.

For the main-sequence stars, no accurate spectral types are available and the measured $B-V$ colors turn out to be anomalously red (Arp 1959b; Robertson 1974; Bessell 1991b). This circumstance renders very hazardous any attempt to estimate effective temperatures and bolometric corrections. A number of explanations of the reddening have been offered (Robertson 1974; Garmany, Conti, & Massey 1987; Bessell 1991b), but as a fourth possible explanation we note that most of the upper main-sequence stars in NGC 330 are emission-line B stars (Feast 1972), which in our Galaxy at least are significantly redder than ordinary B stars (Schmidt-Kaler 1964; Schild 1966). In the present state of uncertainty, it is safe only to make use of the visual apparent magnitude of the tip of the main-sequence turnup—a constraint accepted also by Robertson (1973) and CJF.

4.2. *Theoretical Interpretation*

4.2.1. *Criterion for Convection and Semiconvection*

The evolved blue and red supergiants of NGC 330 are plotted on the theoretical H-R diagram in Figure 4. Their average luminosity suggests a characteristic mass of $\sim 12\ M_\odot$ if convective core overshooting is not too large. In view of the significant number of observed red supergiants, it is possible to rule out the Schwarzschild criterion for convection and semiconvection, because this criterion causes metal-poor stellar models of $\sim 12\ M_\odot$ to burn all but the very last of their core helium in the blue supergiant region owing to a combination of low envelope opacities and large-scale convective homogenization of the intermediate layers of the star. Robertson (1973) was unable to reach a decision about the correct criterion for convection and semiconvection because he employed stellar models with a solar metallicity, which become red supergiants for a significant amount of time during core helium burning.

Barbuy et al. (1991) have suggested that the normal N/C ratios seen in the two red supergiants they studied in this cluster may indicate that the red supergiants are just now entering the red phase, before deep convective envelope mixing dredges up to the surface the products of former hydrogen burning, including N from the CNO bi-cycle. However, this approach stage is very rapid, and it is unlikely that any observed red supergiants would be found in it. Rather, the observed stars are, almost certainly, already stabilized red supergiants. According to our models, metal-poor red supergiants in a settled nuclear-burning state occur only when the Ledoux criterion is adopted, and have surprisingly shallow convective envelopes, which never penetrate down into the hydrogen-processed layers and so never significantly perturb the N/C ratio at the stellar surface. When these stars subsequently evolve into blue supergiants, their surface N/C ratios remain normal. Interestingly, Barbuy et al. (1991) have also detected a normal N/C ratio for the one blue supergiant that they examined. Consequently, there is no sustainable evidence in favor of the Schwarzschild criterion to be derived from the observed supergiants in NGC 330.

It is worth mentioning here that most of the available observational evidence concerning supergiants of high mass in our Galaxy and in the Large Magellanic Cloud also tends to disfavor the Schwarzschild criterion, although this evidence has

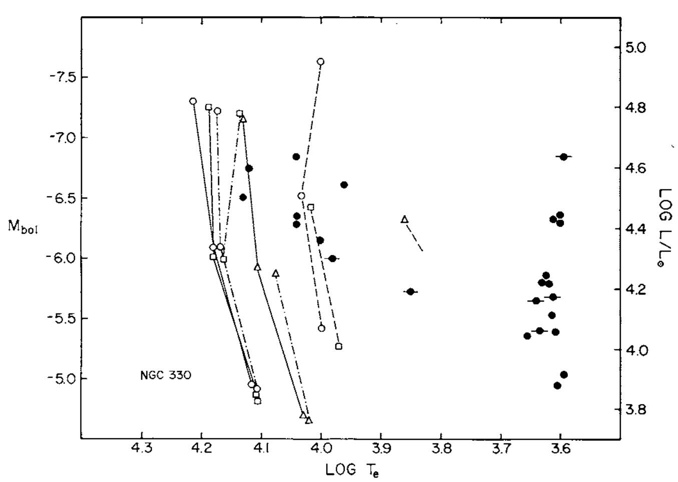

FIG. 4.—H-R diagram, in theoretical coordinates, for the evolved supergiants in NGC 330. Dots with bars indicate stars whose effective temperatures were derived only from $B-V$ colors. Theoretical tips of the blue loops are shown for three cases: Cox-Stewart opacities with $Z_e = 0.004$ (*circles*); Rogers-Iglesias opacities with $Z_e = 0.002$ (*squares*); and Rogers-Iglesias opacities with $Z_e = 0.004$ (*triangles*). Solid lines refer to stellar models without convective core overshooting, and dashed lines refer to stellar models with overshooting parameterized by $d/H_p = 0.35$. Dash-dot lines refer to nonovershooting stellar models with mass loss included.

never been conclusive before (Stothers & Lloyd Evans 1970; Stothers 1975; Stothers & Chin 1976; Fitzpatrick & Garmany 1990).

4.2.2. *Convective Core Overshooting*

Theoretical blue edges that represent the tips of blue loops during the core helium-burning phase are shown in Figure 4 for our metal-poor stellar models. There is a remarkably slight dependence of the blue edges on the choice of opacities, initial metals abundance, or rate of mass loss, but a very strong dependence on convective core overshooting. Since possible errors in log T_e for the observed blue supergiants are not expected to exceed ± 0.03 dex (Stothers 1991) and since these nine stars obviously merely sample the zone of possible occupation by blue supergiants, it is clear that the observations of NGC 330 are consistent with the assumption of no overshooting. A conservative upper limit of $d/H_P < 0.2$ can be set. This upper limit should hold even if the stars experience fast interior rotation (compare Kippenhahn et al. 1970; Meyer-Hofmeister 1972). Only deep overshooting from the outer convection zone could counteract the reduction of the blue loop caused by extensive convective core overshooting (Alongi et al. 1991). This possibility seems unlikely, though, in view of the result of the following test, which is independent of the extent of downward convective envelope overshooting.

As judged from their luminosities, the blue and red supergiants appear to be about equally dispersed in age (allowing for some possible intrinsic variability of the red supergiants). It follows that the mean ages, and hence the mean masses, in the two groups of stars are probably very nearly the same. On evolutionary grounds, this circumstance would be expected in any case for a sufficiently large sample of supergiants in a single cluster, as here. Accordingly, it is meaningful to interpret the observed difference between the average luminosity of the blue supergiants and the average luminosity of the red supergiants as a purely evolutionary effect. This difference is $\Delta\langle M_{\rm bol}\rangle = -0.60 \pm 0.18$.

The bolometric corrections are the only likely source of uncertainty affecting $\Delta\langle M_{\rm bol}\rangle$. Using 10% as the possible error in the effective temperatures and intercomparing the published bolometric correction scales for massive supergiants (Johnson 1966; Code et al. 1976; Flower 1977; CJF), we estimate that accidental errors in the assigned bolometric corrections for supergiants in NGC 330 are not likely to exceed ± 0.15 mag, while systematic errors should be much smaller. The reason is very simply that the bolometric corrections themselves range from zero to only a few tenths of a magnitude for these stars. This implies that $\Delta\langle M_{\rm bol}\rangle$ should be well determined observationally.

Theoretical predictions of $\Delta\langle M_{\rm bol}\rangle$ are practically insensitive to all of the free stellar parameters discussed or mentioned in § 2, except for core mass and, to a lesser extent, total mass. Figure 5 shows our theoretical results for 12 $M_\odot$. Since these predictions depend very little on initial chemical composition (see also Chin & Stothers 1991), it is easy to check them against other authors' results for stars in the mass range 9–15 $M_\odot$ with a low to solar metallicity. Thus, $\Delta\langle M_{\rm bol}\rangle$ is found to be -0.4 ± 0.1 for $d/H_P = 0$ (Becker & Cox 1982; Bertelli, Bressan, & Chiosi 1985; Maeder 1987) and 0.0 ± 0.1 for $d/H_P = 0.25$–0.35 (Becker & Cox 1982; Bertelli et al. 1986; Maeder 1990). Note that the cluster observations are fully consistent with no overshooting at all, especially if one also allows for the fact that fast interior rotation of the stars is capable of

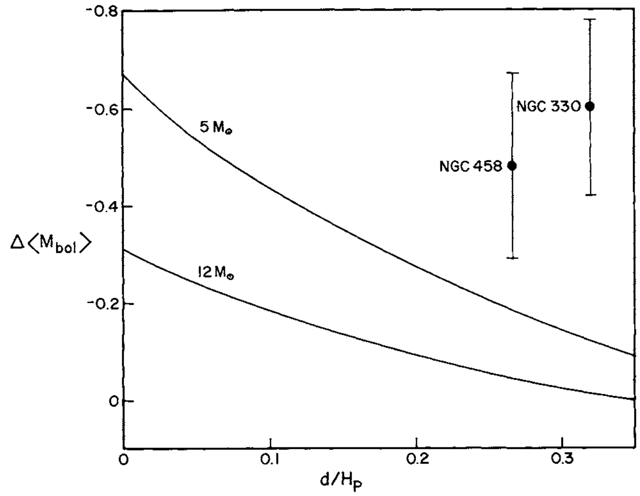

FIG. 5.—Average bolometric magnitude difference between the hot stars and the cool stars evolving during the core helium-burning phase. Observed points for NGC 330 and NGC 458 should be compared with the corresponding theoretical predictions.

making $\Delta\langle M_{\rm bol}\rangle$ more negative by as much as -0.15 (Kippenhahn et al. 1970; Meyer-Hofmeister 1972). At the 2 σ level, a realistic limit on the overshoot parameter can be placed at $d/H_P < 0.2$.

The top of the main-sequence turnup in NGC 330 contains the most massive main-sequence stars currently in the cluster. Similarly, the brightest evolved blue supergiants in this cluster are the most massive of their class. Since the process of formation of massive stars undoubtedly continued for some time, it is reasonable to assume, as a rough approximation, that the masses of the most massive existing members of both groups are identical. Then the observed difference of their visual magnitudes, $\Delta V = -1.7$ mag, which needs no correction for duplicity because binaries are mostly absent, can be simply compared with the theoretical magnitude difference between stars burning core helium and stars of the same mass at the end of the core hydrogen-burning phase. With or without the presence of convective core overshooting, the predicted value of ΔV for stars of $\sim 12\ M_\odot$ is -2.5 mag for either set of opacities. However, a better match with observations could be achieved if the metals abundance were solar. CJF reached the same conclusion by comparing the mean visual magnitude of the group of red supergiants with the mean visual magnitude of stars at the top of the main-sequence turnup. The principal reason for a composition dependence of this sort is that a metal-rich star lying at the top of the turnup has a lower effective temperature, and hence a smaller bolometric correction, than does a metal-poor star. A similar effect could be produced by rotational lowering of the effective temperature, which may be relevant here because the turnup in this cluster contains many anomalously red Be stars, which are commonly believed to be fast rotators.

In NGC 330, the ratio of the numbers of blue and red supergiants is observed to be $n_b/n_r = 0.6$. There is no need to "correct" this ratio for the possible effects of binary star interaction because close binary systems are conspicuously rare or absent among the brightest stars in this cluster (Feast & Black 1980). Therefore, it is possible to compare n_b/n_r directly with the theoretically predicted ratio of lifetimes, τ_b/τ_r. In the absence of convective core overshooting, τ_b/τ_r turns out to be

~ 6 for the Cox-Stewart opacities with $Z_e = 0.004$, ~ 5 for the Rogers-Iglesias opacities with $Z_e = 0.002$, and ~ 1.2 for the latter opacities with $Z_e = 0.004$. The last case is marginally consistent with the observed n_b/n_r if possible statistical fluctuations of n_b and n_r are allowed for. On the other hand, evidence using n_b/n_r is weak for purely theoretical reasons in that τ_b/τ_r can display considerable sensitivity to variations in other physical input parameters in the stellar models (Chin & Stothers 1991). It would be interesting to know how n_b/n_r changes with interior angular momentum.

4.2.3. *Age of the Cluster*

A determination of the true dispersion of stellar masses, and hence ages, among the supergiants of NGC 330 depends on how much of the observed luminosity spread arises from evolutionary effects and how much from mass differences. From the results of the preceding section, we may ignore any significant convective core overshooting. Then, for a low metallicity, the luminosity spread can only be interpreted as being due, almost wholly, to a dispersion of masses, 9–14 $M_\odot$ (Figs. 1–3). This mass range is easily translated into a range of ages, (1.5–3.0) $\times 10^7$ yr. We conclude that the age of NGC 330 probably lies around 3×10^7 yr.

In earlier studies, Robertson (1973) roughly inferred a mean supergiant mass of 15 $M_\odot$, and CJF claimed a range of masses of 13–27 $M_\odot$ for the blue supergiants and 12–19 $M_\odot$ for the red supergiants. These values are much too large, even allowing for the greater distance to the Small Magellanic Cloud that these authors adopted. Not surprisingly, most of the previously derived ages for NGC 330 correspondingly small: 1.2×10^7 yr (Robertson 1972, 1973), 0.7×10^7 yr (Hodge 1983), 1.2×10^7 yr (CJF), and 1.0×10^7 yr (Mateo 1988). However, Hodge (1984) later revised his previous age estimate to 2.0×10^7 yr.

5. ANALYSIS OF THE GIANTS IN NGC 458

5.1. *Observational Data*

No detailed observational data other than Arp's (1959a) B and V photometry are available for NGC 458. Since this intermediate-age cluster lies in the outskirts of the Small Magellanic Cloud, its internal reddening can be assumed to be negligible; for the foreground reddening, we adopt $E_{B-V} = 0.03$, which is a weighted mean of the values given by McNamara & Feltz (1980) and Bessell (1991a). Arp's data can be converted to theoretical quantities by using the same transformations that we employed for the stars in NGC 330, except that for the red giants in NGC 458 Flower's (1977) relations between $B-V$, effective temperature, and bolometric correction are adopted, because $J-K$ colors are not available.

It is difficult to estimate how accurate Arp's $B-V$ colors are. Stars at the top of the main-sequence turnup in NGC 458 show a total scatter of up to ± 0.15 mag around a mean uncorrected $B-V$ color of -0.10, but these stars are somewhat fainter than the bluest evolved giants, which lie in the apparent magnitude range $15 < V < 17$. Judging from the comparison given by CJF between their own color indices and those of Arp (1959b) and Robertson (1974) for the bright stars with $V \leq 16$ in NGC 330, we estimate that Arp's $B-V$ colors for the evolved giants in NGC 458 could be in error by up to ± 0.10 mag among the blue and yellow giants and by up to ± 0.05 mag among the red giants. Such errors would have little effect on the adopted bolometric corrections which are in any case small. But they could produce errors in log T_e of ± 0.12 dex for the bluest evolved giants, although much less for the cooler yellow and red giants. The main-sequence stars, being even less accurately observed, can be used here only for establishing the tip of the main-sequence turnup.

5.2. *Theoretical Interpretation*

5.2.1. *Convective Core Overshooting*

Despite the large uncertainty attached to the derived effective temperatures of the bluest giants in NGC 458, the positions of all the giants on the theoretical H-R diagram (Fig. 6) are known at least well enough to set a useful upper limit on the amount of convective core overshooting. Based on the plotted theoretical blue edges, this limit is $d/H_P < 0.2$, although the assumption of no overshooting at all satisfies well the observations of these many evolved stars of $\sim 5\ M_\odot$.

The hot and cool giants differ in mean luminosity by $\Delta\langle M_{\rm bol}\rangle = -0.48 \pm 0.19$, with almost none of the uncertainty coming from the bolometric corrections. According to Figure 5, this difference implies an upper limit of $d/H_P < 0.35$, at the 2 σ level. However, the data are completely consistent with the assumption of no overshooting at all.

Between the brightest of the hot evolved giants and the brightest stars on the main-sequence turnup lies an observed separation of $\Delta V = -1.7$ mag. The theoretically predicted gap for a star of $\sim 5\ M_\odot$ is -2.3 mag for the Cox-Stewart opacities and -1.8 mag for the Rogers-Iglesias opacities (these values are not significantly dependent on convective core overshooting). The Rogers-Iglesias opacities are clearly preferred.

NGC 458 altogether contains 10 hot and 22 cool giants. The observed number ratio, n_b/n_r, is therefore nearly 0.5. Without overshooting, the theoretically predicted ratio, τ_b/τ_r, is ~ 2 for the Cox-Stewart opacities with $Z_e = 0.004$, ~ 2 for the Rogers-Iglesias opacities with $Z_e = 0.002$, and ~ 0.5 for the latter opacities with $Z_e = 0.004$. Although the evidence is not very firm in view of the other possible uncertainties in the stellar models (Chin & Stothers 1991), it appears that the Rogers-Iglesias opacities are again favored.

Finally, the absence of any superluminous red giants in NGC 458 constitutes evidence for an acceleration of evolution

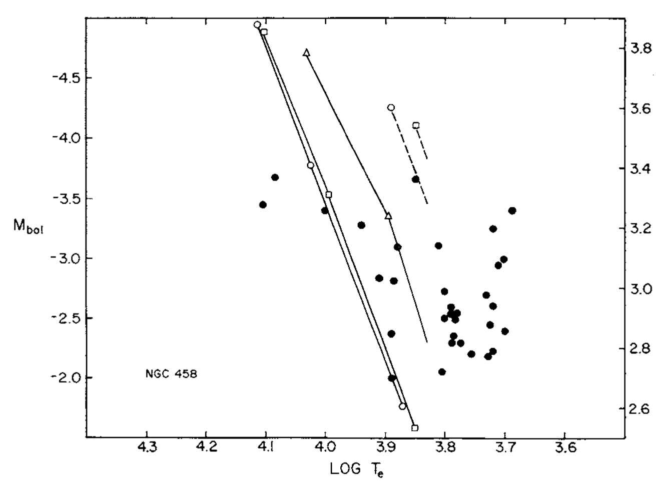

FIG. 6.—H-R diagram, in theoretical coordinates, for the evolved giants in NGC 458. All the effective temperatures shown were derived only from $B-V$ colors. Theoretical tips of the blue loops are plotted in the same notation as for Fig. 4.

under the influence of neutrino emission after core helium burning (Stothers 1969).

5.2.2. *Age of the Cluster*

For any low metals abundance, the range of present giant masses in NGC 458 must be 4–5 $M_\odot$ and the corresponding dispersion of ages is (8–13) $\times 10^7$ yr. In an early study, Hayashi, Nishida, & Sugimoto (1962) derived a mean giant mass of 4 $M_\odot$ and an age of 8×10^7 yr. Hodge's (1983) age for the cluster is much smaller, $(5 \pm 1) \times 10^7$ yr. The true age is probably not far from 1×10^8 yr.

6. CONCLUSION

Four different comparisons between observed stars and theoretical models have been made for the brightest stars in the blue populous clusters NGC 330 and NGC 458 of the Small Magellanic Cloud. All of these comparisons utilize either effective temperatures or relative luminosities, and so do not depend in any important way on knowing the precise distance of the Small Cloud.

Two of the comparisons, involving the highest effective temperature of the hot evolved stars and the ratio of luminosities of the hot and cool evolved stars, constrain the convective core overshooting parameter to a limit of $d/H_P < 0.2$, independently of any uncertainty about the stars' initial chemical composition. The observational data are actually consistent with $d/H_P = 0$. Although the maximum effective temperature can be shifted to somewhat higher values under certain extreme conditions such as very deep convective envelope overshooting and critically fast interior rotation, the luminosity ratio appears to be relatively insensitive to all of the physical uncertainties except the uncertain core mass. These results place a severe restriction on theories that predict, or at least allow, extensive convective core overshooting. By implication, deep convective envelope overshooting is now also highly improbable.

The two other comparisons, based on the ratio of the numbers of hot and cool evolved stars and the ratio of the luminosities of the hot evolved stars and the brightest main-sequence stars, suggest that the new Rogers-Iglesias opacities are superior to the standard Cox-Stewart opacities. However, neither of these comparisons in entirely satisfactory for NGC 330; fast interior rotation of the stars in this cluster may be responsible for the remaining discrepancies.

Both of our main conclusions, a limit of $d/H_P < 0.2$ and the superiority of the Rogers-Iglesias opacities, have also been reached in the case of stars of similar age in our Galaxy (Stothers 1991; Stothers & Chin 1991b). In addition, we have been able to show, with some certainty, that the Ledoux criterion should be used for the outbreak of convective motions and for the condition of convective neutrality in the semiconvective zones of massive stars.

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